ISC 5935 - Computational Issues in Optimization (Spring 2009)

Instructor: Prof .I. M. Navon Hours: 12:20 PM -1:10 PM

Recommended Book: <u>Numerical Optimization (Springer Series in Operations Research</u> <u>and Financial Engineering)</u> by Jorge Nocedal and Stephen Wright (Hardcover - Jul 27, 2006)

- Hardcover: 664 pages
- Publisher: Springer; 2nd ed. edition (July 27, 2006)
- Language: English
- ISBN-10: 0387303030

SYLLABUS

- Introduction
- •The NCP problem
- Characterization of single variable unconstrained minimum

• Necessary and sufficient first and second order conditions for an unconstrained minimum.

- Examples
- Unconstrained minimization methods
- · Properties of a quadratic function
- Convex and concave functions
- Method of bisection
- Newton's method
- The secant method
- Regula -falsi method
- Univariate minimization
- Unimodal functions
- · Fibonnacci search for univariate minimum and algorithm
- · Golden section method and algorithm
- Quadratic fit-polynomial interpolation for univariate minimum
- Cubic fit polynomial interpolation
- Methods for minimization of multivariate smooth functions
- Direction of descent
- Global convergence of descent methods

- · Computing step-length along direction of descent
- Armijo rule for step length
- Goldstein Armijo test for step length
- Trust region methods (new)
- The Wolfe test for step length
- Step length for a quadratic function
- · Computation of good search directions
- Method of steepest descent
- Examples
- · Newton's method for multivariate functions
- Problems with Newton's method
- · Modification of Newton's method for indefinite Hessian
- · Methods based on Choleski decomposition

• Quasi-Newton Methods

- Matrix outer products and rank one matrices
- Symmetry and positive-definiteness in Hessian approximations with Q-N method.
- · Algorithmic implementation of Quasi-Newton methods
- Davidon Fletcher Powell (DFP) Quasi-Newton (Q-N) Algorithm
- Examples
- Positive definiteness of DFP Q-N update to the Hessian.
- Finite-step convergence of DFP Q-N method
- Davidon Fletcher Powell Q-N method with cubic interpolation for the step-length.
- Q-N method for positive definite quadratic form
- Variable metric form of the Quasi-Newton descent methods.
- Broyden-Fletcher-Goldfarb-Shanno (BFGS) Q-N method
- Use of Sherman Morrisson Inverse matrix formula
- Variational metric properties of DFP and BFGS Q-N Methods.
- Scaling issues related to Quasi-Newton method
- Eigenvalues of Hessian and the interlocking eigenvalues Theorem.

• Choice of appropriate scale factors and self scaling Q-N algorithm.

- Limited Memory Quasi-Newton Methods
- Conjugate Gradient algorithms
- Conjugate directions Theorem
- Expanding Subspace Theorem

- Geometric 2-D interpretation of conjugacy concept and polarity.
- Conjugate gradient (C-G) algorithm and examples.
- C-G Theorem
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- Fletcher -Reeves C-G algorithm for nonlinear functions.
- C-G methods viewed as an optimal process.
- Convergence rate of the C-G method
- Hessian with clustered eigenvalues and preconditioning
- Examples
- Different C-G algorithms with various line-search methods
- Polak-Ribiere C-G algorithm
- Restart methods for nonlinear C-G algorithm
- Powell's (1977) restart C-G method using Beale's condition
- Memoryless Q-N like C-G methods: Codes and algorithms.
- Scaling and preconditioning in C-G methods

• Variable-storage Q-N like C-G Methods : Codes, Algorithms and Examples

• Truncated Newton methods and examples

• Use of scientific minimization software libraries. (NAG, IMSL, NEOS)

Constrained Optimization

- First and second order conditions : theory
- Karush- Kuhn Tucker (KKT) Theorem
- Linear and nonlinear constraints and the active set approach
- Penalty and barrier functions methods
- Reduced gradient and gradient projection methods
- Exact penalty methods
- Augmented Lagrangian Methods
- Sequential Quadratic Programming (SQP)
- Reduced Gradient methods (GRG)
- Rate of convergence of constrained minimization methods
- Nondifferentiable constrained minimization methods

• The subgradient bundle algorithm for nondifferentiable minimization

- Global Minimization methods
- Simulated annealing methods theory and applications
- Genetic algorithms-Theory and applications
- Other global search methods

• Experience with simulated annealing and genetic algorithms